Notes on Gravity Wave Sign Conventions in the UM
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May 2002

In producing the new gravity-wave code for the UM, it has become clear that there are some difficulties with the sign conventions in use. In particular, it is possible that the UM has the right answer for the wrong reasons.

In the code (eg `gwsatn3a.f`) the accelerations are calculated using an expression like

```
DU_DT(I,K-1) = G*(X_STRESS(I,KL) - X_STRESS(I,KU))/DELTA_P
```

where `DU_DT` is obviously the acceleration, and the `KL` and `KU` indicate the lower and upper layers respectively. `DELTA_P` is the pressure difference across the layers. This translates to

$$\Delta u = g\frac{(\tau_z - \tau_{z+\Delta z})}{\Delta p} \tag{1}$$

which, when combined with the timestep, gives us

$$\frac{\partial u}{\partial t} = -g\frac{\partial \tau}{\partial p}, \tag{2}$$

where the minus sign arrives because we subtract the lower layer value from the one above. If we make use of the hydrostatic equation:

$$\frac{\partial p}{\partial z} = -\rho g$$

then we can rewrite equation 2 as

$$\frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial \tau}{\partial z}, \tag{3}$$

which is a problem, since we expect a minus sign on the right hand side (RHS) of this equation

Why do we expect a minus sign on the RHS? It all comes down to a definition of the stress. In terms of wave horizontal momentum, the expression can be writen as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial}{\partial z}(\rho\overline{u\prime w\prime}), \tag{4}$$

where the expression $\rho\overline{u\prime w\prime}$ is the upward flux of eastward momentum (or more formally the flux of pseudomomentum).

In the UM, the surface stress is defined as $\tau = \frac{1}{2}k\rho_s UNH^2$ where H^2 is an orographic variance. However, what is not obvious from any of the Met Office papers is that this stress is actually defined as a drag (but see Lott, 1995 who does make this explicit). That is, $\tau = -\rho\overline{u\prime w\prime}$. This means there is an implicit minus sign in the formula for surface stress, which is then compensated for by a missing minus sign in the expression for the acceleration!